

Adaptation amongst Heterogeneous Interacting Agents in a model Economy

Anirban Chakraborti ¹

Marko Sysi-Aho, Kimmo Kaski

Laboratory of Computational Engineering,

Helsinki University of Technology,

P. O. Box 9203, FIN-02015 HUT, Finland.

Email: anirban@lce.hut.fi

Abstract

The adaptive dynamics of heterogeneous interacting agents competing for scarce resources are believed to underlie the behaviour of most economic systems. The economic agents have to be the best in order to survive in the market. In recent studies of market behaviour, tools of statistical physics have been combined with theories of economics, like game theory, which deals with making decisions when a number of rational opponents are involved under conditions of conflict and competition. Here, we consider a simple game theoretical model of heterogeneous interacting agents and assume that the agents play the “basic minority game” and in addition, the agents adapt by modifying their strategies periodically, depending on their performances, in order to be the best. The strategies are modified using genetic crossover, inspired from genetic evolution in biology. We find interesting patterns in the performances of the agents when they use these genetic-crossover strategies and the game enters into more “efficient” states.

¹A.C. presented a paper with similar contents, entitled “Biology helps you to win a game”, at the conference “Unconventional Applications of Statistical Physics” held at Saha Institute of Nuclear Physics, Kolkata on March 20-22, 2003.

1 Introduction

Is the “survival of the fittest” principle limited to biology only? Perhaps not and there could be other spheres of life in which this principle is applicable. Competition plays a key role and in order to compete and thus survive in any environment or situation, one primarily needs to adapt in order to succeed. Then what is adaptation and evolution? Adaptation is an alteration or adjustment in structure or habits, often hereditary, by which a species or individual improves its condition in relationship to its environment. Evolution is the change in the genetic composition of a population during successive generations, as a result of natural selection acting on the genetic variation among individuals, and resulting in the development of a new species. Here, we show that in the behaviour of various complex systems found in natural and social environments [1, 2, 3, 4, 5], that can be characterized by the competition among interacting agents for scarce resources, adaptation to the environment plays a very important role.

These agents could be diverse in form and in capability, ranging for example, from carcinogenic cells in the human body to multinational firms in the global financial market. In these dynamically evolving complex systems the nature of agents and their behaviour differ a lot but they have a common underlying mechanism. In order to have a deeper understanding of the interactions of the large number of agents, one should first consider the individual capabilities of the agents. Its behaviour may be thought of as a collection of simple rules governing “responses” to numerous “stimuli”. The rules of action serve as the agents’ strategies, and the behaviour of an agent is the rules acting sequentially. Therefore, in order to model any complex dynamically adaptive system, a major concern is the selection and representation of the stimuli and responses, since the behaviour and strategies of the component agents are determined thereby. Then the agent needs to adapt to different situations, where the experience of an agent guides it to change its structure so that as time passes, the agent learns to make better use of the environment for its own benefit. However, the timescales over which the agents adapt vary from one individual to another and also from one system to another.

In complex adaptive systems, many interesting temporal patterns are produced, since a major part of the environment of a particular agent includes other adaptive agents and a considerable amount of agent’s effort goes in adaptation and reaction to the other agents. Thus the situation is considerably different and more complicated than in game theory [6] and conventional

theories in economics, where the study is of patterns in behavioural equilibrium that induce no further interaction.

In this paper, we study a simple game based on the basic minority game [7, 8, 10, 9, 11], where the agents adapt themselves by modifying their strategies from time to time, depending on their current performances, using genetic crossover mechanisms [12, 13, 14, 15]. The game can be a very simple representation of a complex adaptive system. We make a comparative study of their performances with the various mechanisms and in a “test” situation.

2 Model

In this section we give a brief description of the model. The basic minority game consists of an odd number N of agents who can perform at a given time t , any of the two possible actions denoted here by 0 or 1. The minority game was based on the El Farol bar problem, created by Brian Arthur, in which a population of agents have to decide whether to go to the bar every Thursday night, and so there were two possible actions “to attend” denoted by 1 and “not to attend” denoted by 0, depending on whether the bar was too crowded or not [5]. An agent wins the game if it is one of the members of the minority group. All the agents are assumed to have access to finite amount of “global” information: a common bit-string “memory” of the M most recent outcomes. With this there are 2^M possible “history” bit-strings. Now, a “strategy” consists of two possible responses, which in the binary sense are an action 0 or action 1 to each possible history bit-strings. Thus, there are 2^{2^M} possible strategies constituting the whole “strategy space”. Each player has an initial pool of k strategies drawn randomly from the whole strategy space.

Each time the game has been played, time t is incremented by unity and one “virtual” point is assigned to the strategies that predicted the correct outcome and the best strategy is the one which has the highest virtual point score. The performance of a player is measured by the number of times the player wins, and the strategy, which the player uses to win, gets a “real” point. The number of agents who have chosen a particular action, say 1 which represents “to attend”, is denoted by $A_1(t)$ (also referred as “attendance”) and it varies with time. We have plotted the attendance and performance for the basic minority game in Fig. 1.

Now we define the total utility of the system as the number of persons in

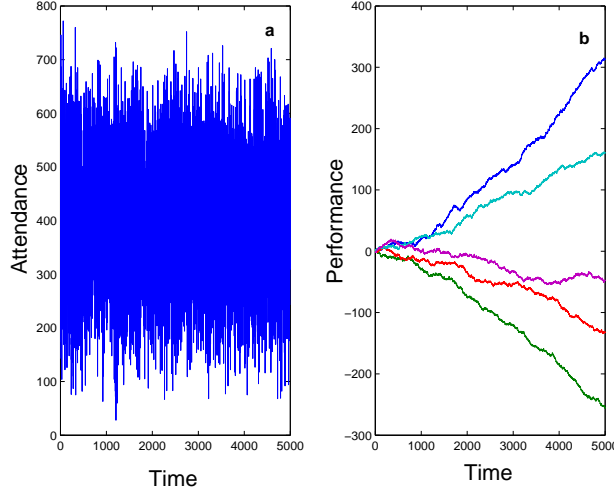


Figure 1: Plots of (a) attendance and (b) performance of the players for the basic minority game with $N = 801$, $M = 6$, $k = 10$ and $T = 5000$.

the minority group at a given time t . For convenience, we mathematically define a scaled utility (total utility/maximum utility) as

$$U = [(1 - \theta(x_t - x_M))x_t + \theta(x_t - x_M)(N - x_t)]/x_M, \quad (1)$$

where $x_M = (N - 1)/2$, x_t is either equal to $A_1(t)$ or $A_0(t)$, and $\Theta(x)$ is Heaviside step function:

$$\theta(x) = \begin{cases} 0 & \text{when } x \leq 0 \\ 1 & \text{when } x > 0. \end{cases}$$

The players examine their performances after every time interval τ . If a player finds that he is among the fraction n (where $0 < n < 1$) who are the worst performing players, he adapts himself and modifies his strategies. The mechanism by which the player creates new strategies is genetic crossover, whereby he selects the two “parents” from his pool of k strategies and creates two new “children” [14, 15], as described in Fig. 2.

If the parents are chosen randomly from the pool of strategies then the mechanism represents a “one-point genetic crossover” and if the parents are the best strategies then the mechanism represents a “hybridized genetic crossover”. The children may replace parents or two worst strategies

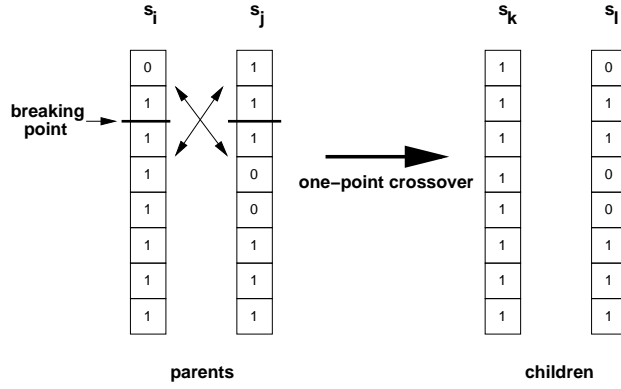


Figure 2: Schematic diagram to illustrate the mechanism of one-point genetic crossover for producing new strategies. The strategies s_i and s_j are the parents. We choose the breaking point randomly and through this one-point genetic crossover, the children s_k and s_l are produced.

and accordingly four different interesting cases arise: (a) one-point genetic crossover with parents “killed”, i.e. parents are replaced by the children, (b) one-point genetic crossover with parents “saved”, i.e. the two worst strategies are replaced by the children but the parents are retained, (c) hybridized genetic crossover with parents “killed” and (d) hybridized genetic crossover with parents “saved”.

It should be noted that the mechanism of evolution of strategies is considerably different from earlier attempts [7, 16, 17]. This is because in this mechanism the strategies are changed by the agents themselves and even though the strategy space evolves continuously, its size and dimensionality remain the same.

The Hamming distance d_H between two bit-strings is defined as the ratio of the number of uncommon bits to the total length of the bit strings. It is a measure of the correlation between two strategies:

$$d_H = \begin{cases} 0 & \text{correlated} \\ 0.5 & \text{uncorrelated} \\ 1 & \text{anti-correlated} \end{cases}$$

which can be plotted as the game evolves.

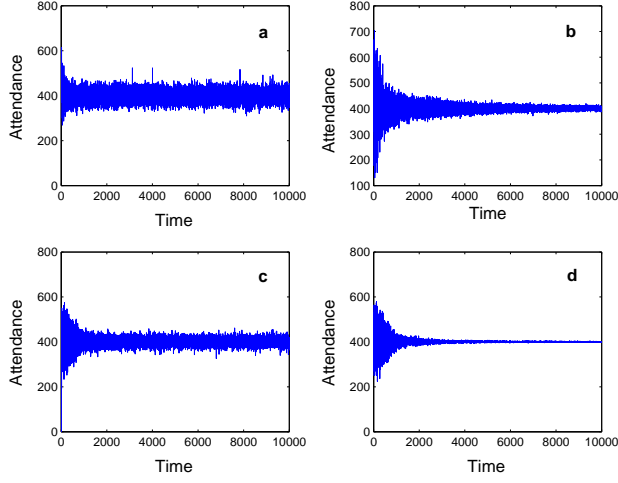


Figure 3: Plots of the attendances by choosing parents randomly (a) and (b), and using the best parents in a player’s pool (c) and (d). In (a) and (c) case parents are replaced by children and in (b) and (d) case children replace the two worst strategies. Simulations have been done with $N = 801$, $M = 6$, $k = 16$, $\tau = 40$, $n = 0.4$ and $T = 10000$.

3 Results

In order to determine which mechanism is the most efficient, we have made a comparative study of the four cases, mentioned earlier. We plot the attendance as a function of time for the different mechanisms in Fig. 3.

In Fig. 4 we show the total utility of the system in each of the cases (a)-(d), where we have plotted results of the average over 100 runs and each point in the utility curve represents a time average taken over a bin of length 50 time-steps. The simulation time is doubled from those in Fig. 3, in order to expose the asymptotic behaviour better. On the basis of Figs. 3 and 4, we find that the case (d) is the most efficient.

In Fig. 5 (a) one can see the evolution of the average Hamming distance of all the strategies of a player in a game, where the player adapts using one-point genetic crossover and the two worst strategies are replaced by the children and the parents are also saved. It should be noted that the Hamming distance can change only when the worst strategies are replaced by the children and the parents are saved, where the bits in a strategy pool can

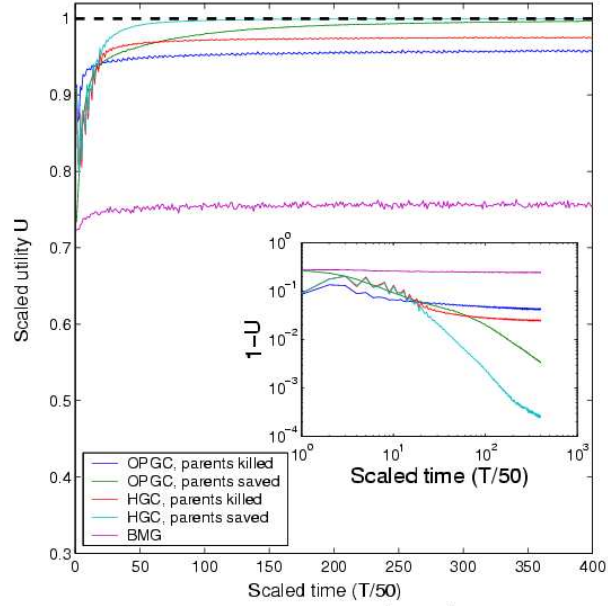


Figure 4: Plots of the scaled utilities of the four different mechanisms in comparison with that of the basic minority game. Each curve represents an ensemble average over 100 runs and each point in a curve is a time average over a bin of length 50 time-steps. In the inset, the quantity $(1 - U)$ is plotted against scaled time in the double logarithmic scale. Simulations are done with $N = 801$, $M = 6$, $k = 16$, $\tau = 40$, $n = 0.4$ and $T = 20000$.

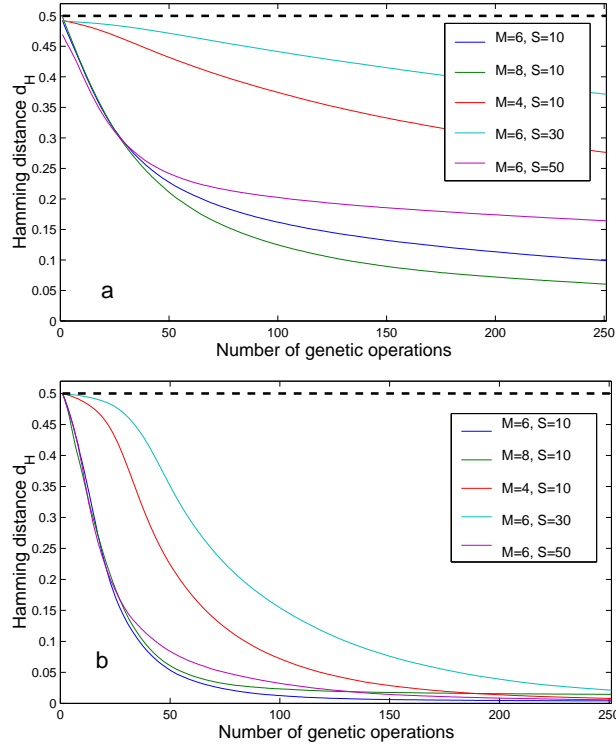


Figure 5: Plot of the average Hamming distance of all the strategies in a pool of a player with time, where the player adapts using (a) one-point genetic crossover and (b) hybridized genetic crossover, and in both cases the two worst strategies are replaced by the children and the parents are also saved. Each curve is an ensemble average over 20 runs.

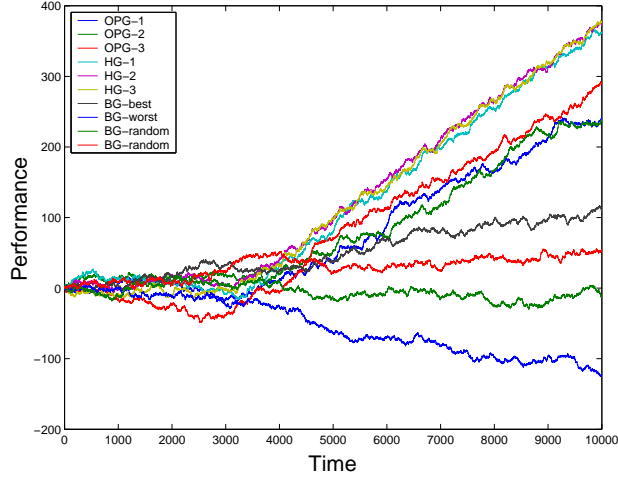


Figure 6: Plot of the performance of the players where after $T = 3120$ time-steps, six players begin to adapt and modify their strategies: three using hybridized genetic crossover mechanism and the other three using one point genetic crossover, where children replace the parents. Other players play the basic minority game all the time and do not adapt. The simulations are done with $N = 801$, $M = 8$, $k = 16$, $n = 0.3$, $\tau = 80$, and $T = 10000$.

change over time. Otherwise the bits in the pool of strategies remain the same. We observe that the curves tend to move downwards from around 0.5 towards zero, which means that as the time evolves, the correlation amongst the strategies increases and the strategies in the pool of a particular agent converges towards one strategy. The nature of the curves depend a lot on the parameters of the game. In Fig. 5 (b) one can see the evolution of the average Hamming distance of all the strategies of a player in the game, where the player adapts using hybridized genetic crossover and the two worst strategies are replaced by the children and the parents are also saved. Here too, the strategies in the pool of a particular agent converges towards one strategy, and at a faster rate than with the previous mechanism. We observe that increasing memory M does not change dramatically the convergence rate, but as we increase the number of strategies in the pools, the convergence slows down.

In order to investigate what happens in the level of an individual agent, we created a competitive surrounding- “test” situation where after $T = 3120$

time-steps, six players begin to adapt and modify their strategies such that three are using hybridized genetic crossover mechanism and the other three one point genetic crossover, where children replace the parents. The rest of the players play the basic minority game. In this case it turns out that in the end the best players are those who use the hybridized mechanism, second best are those using the one-point mechanism, and the bad players those who do not adapt at all (see Fig. 6). In addition it turns out that the competition amongst the players who adapt using the hybridized genetic crossover mechanism is severe.

4 Conclusion

We can summarize our findings by stating that adaptation improves not only the individual player's performance but also improves the total utility of the system. The best results are found for the players who adapt and modify their strategies using the hybridized genetic crossover mechanism and the children replace the two worst strategies and the parents are saved. The mechanism of adaptation is very simple and can be used to model different complex adaptive systems. It can also be potentially developed to include other features like mutation.

5 Acknowledgments

This research was partially supported by the Academy of Finland, Research Centre for Computational Science and Engineering, project no. 44897 (Finnish Centre of Excellence Programme 2000-2005).

References

- [1] G. Parisi, *Physica A* **263**, 557 (1999).
- [2] B. A. Huberman, P. L. T. Pirolli, J. E. Pitkow and R. M. Lukose, *Science* **280**, 95 (1998).
- [3] M. Nowak and R. May, *Nature* **359**, 826 (1992).
- [4] T. Lux and M. Marchesi, *Nature* **397**, 498 (1999).

- [5] W. B. Arthur, *Am. Econ. Rev.* **84**, 406 (1994).
- [6] R. Myerson, *Game Theory: Analysis of Conflict* (Harvard University Press, Cambridge, Massachusetts, 1991).
- [7] D. Challet and Y.-C. Zhang, *Physica A* **246**, 407 (1997).
- [8] D. Challet, M. Marsili and R. Zecchina, *Phys. Rev. Lett.* **84**, 1824 (2000).
- [9] R. Savit, R. Manuca and R. Riolo, *Phys. Rev. Lett.* **82**, 2203 (1999).
- [10] A. Cavagna, J. P. Garrahan, I. Giardina and D. Sherrington, *Phys. Rev. Lett.* **83**, 4429 (1999).
- [11] D. Lamper, S. D. Howison and N. F. Johnson, *Phys. Rev. Lett.* **88**, 17902 (2002).
- [12] J. H. Holland, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor (1975).
- [13] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley, Reading, Massachusetts (1989).
- [14] D. Lawrence (Ed.), *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, New York (1991).
- [15] M. Sysi-Aho, A. Chakraborti and K. Kaski, *preprint available at cond-mat/0209525* (2002); *Physica A*, in press (2003).
- [16] Y. Li, R. Riolo and R. Savit, *Physica A* **276**, 234 (2000).
- [17] Y. Li, R. Riolo and R. Savit, *Physica A* **276**, 265 (2000).